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The Single-year Life Tables in the Developing Countries Mortality Database (DCMD)

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ABSTRACT

Life tables of developing countries are often estimated using a model life table and child (or child and adult) mortality, leaving old-age mortality unchecked and unreliable. Consequently, the number of old-age deaths in the less developed region is the least reliable and, as a matter of fact, also the biggest among various age and development groups. This irony led to establishing the Developing Countries Mortality Database (DCMD) that utilizes observed data on old-age mortality. In the DCMD, multiyear-period life tables are first estimated, and this paper indicates how to convert these period life tables into single-year life tables.

KEYWORDS

Single-year life tables; mortality database; developing countries

1. Introduction

Empirical data used in estimating life tables are collected from deaths (1) registered annually or in census and (2) sampled in surveys. For most developing countries, annual death registrations are either unavailable or unreliable, estimating life tables rely mainly on surveys, and occasionally also on census when it collects information on deaths in the household in the previous 12 months (United Nations Statistics Division 2008).

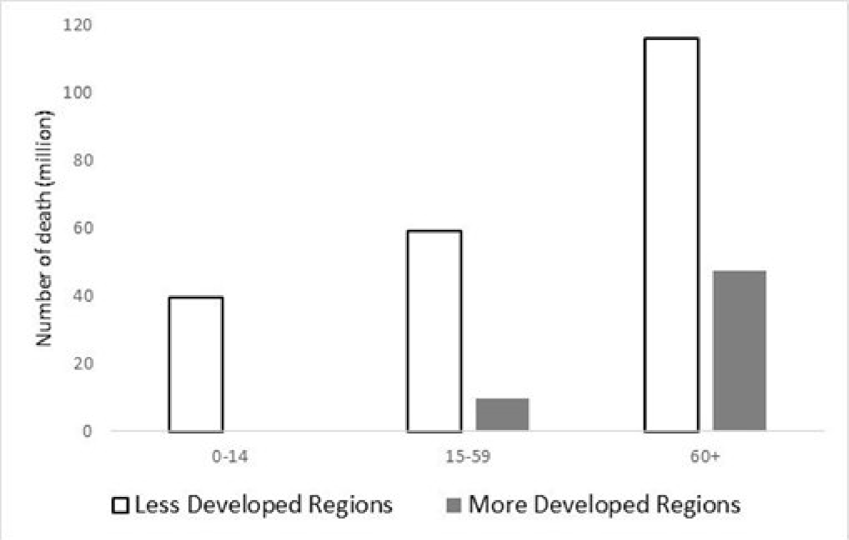
Typical sample surveys often collect information only from a small portion of the population. Subsequently, they cannot directly produce life tables. This is because death rates at some ages, for example 10-20 years, are usually very low and require a large population to be estimated reliably. Nonetheless, sample surveys can provide reliable indicators of mortality for certain age groups when death is not a rare event or when the age group is wide enough.

The most commonly sampled mortality indicator is child mortality, which is the probability of dying between birth and age 5 and is often denoted as ${}_5q_0$. The United Nations Children's Fund (UNICEF, www.childmortality.org) as part of the United Nations Inter-agency Group for Child Mortality Estimation (IGME) has been regularly collecting, analyzing, and publishing child mortality for most of the countries back to the 1970s or earlier. Based on the values of ${}_5q_0$, life tables can be estimated using one-input-parameter model life tables (MLT) such as the Coale-Demeny MLT (Coale, Demeny, and Vaughan 1983), or the United Nations MLT (United Nations Population Division 1982).

Starting from the 1990s, surveys such as the Demographic and Health Surveys (DHS, www.measuredhs.com) have been collecting sibling histories (Timæus 2013), leading to the estimations of adult mortality, which is the probability of dying between age 15 and 60 years, and is denoted as ${}_{45}q_{15}$. Combining data of surveys and other sources (including additional time covariates for countries and periods with missing or deficient data), Wang and colleagues (2012) at the Institute for Heath Metrics and Evaluation (IHME) estimated adult mortality for 187 countries from 1970 to 2010. Utilizing the values of ${}_5q_0$ and ${}_{45}q_{15}$, life tables can be estimated using the two-input-parameter MLT such as the modified logit life table system (Murray et al. 2003) or the log-quadratic model life table with two input parameters (Wilmoth et al. 2012).

Worldwide, the biggest number of deaths among various age and development groups occurred in the less developed regions (Figure 1) in 2010-2015 (United Nations Population Division 2017). Thus, the progress of estimating child and adult mortality, for the reason of more deaths occurring at old ages, should be extended into old ages. The Developing Countries Mortality Database (DCMD) is a project to fulfill this extension. The DCMD aims to provide empirical estimates of mortality at all ages by estimating old-age mortality and, using it together with the estimates of child and adult mortality provided by IGME and IHME, to calculate life tables for all developing countries.

Figure 1. Deaths by age group and development region, 2010-15.



Source: United Nations Population Division, 2017

The DCMD methods include (1) estimating old-age mortality (Li and Gerland 2013), (2) the three-input-parameter MLT (Li 2014) that utilizes child, adult, and old-age mortality to calculate life tables, and (3) the method proposed in this paper.

Applying the DCMD methods to census data of old-age population of the 151 developing countries that have about 100 thousand or more populations in 2015 (United Nations Population Division 2017), multiyear-period life tables for 122 countries are obtained as the preliminary results of DCMD (Li et al. 2018).

This paper proposes the method to convert multiyear-period life tables into single-year life tables, using the relationship between period and single-year estimates and, when available, utilizing also the data on old-age deaths in census years collected from death registration or census. Applying this method, we obtained single-year life tables for the 122 countries, and we provided these life tables at www.lifetables.org. Compared to the reliable life tables from other sources, we show the evidence for the accuracy of these single-year life tables, and we indicate that the DCMD methods could be used to estimate life tables for small populations.

2. Method

2.1. Relationship estimates

In DCMD, the old-age mortality (the probability of dying between ages 60 and 75 years) for a multiyear-period, Qop (where subscript "o" and "p" stands for old age and period, respectively), is estimated using the Census Method that utilizes the numbers of old-age population enumerated in two successive censuses (Li, Mi, and Gerland, 2017), the estimates of child mortality and adult mortality which can be obtained from the IGME (www.childmortality.org) and the IHME (http://www.healthdata.org), and the two-input-parameter (MLT) model life table (Wilmoth et al Sawyer, 2012). After the Qop is estimated for the 122 countries in the DCMD, the corresponding life tables are calculated using the three-input-parameter MLT. The purpose of this paper is to propose the below method that converts Qop into single-year old-age morality, Qo(t), which leads to the calculations of single-year life tables in DCMD.

The two-input-parameter MLT uses child mortality (Qc(t)) and adult mortality (Qa(t)) to produce a life table. Using this life table, old-age mortality, $Qo_2(t)$ (where subscript "2" stands for two-input-parameter MLT), is obtained. The estimates of $Qo_2(t)$ are based on the Qc(t) provided by the IGME and the Qa(t) estimated by the IHME. Thus, the over-time changes in Qo2(t) should contain the effects on Qc(t) and Qa(t) from socioeconomic developments and special events such as natural disasters, epidemics, and violent conflicts that have been taken into account by the IGME and IHME.

The basis of converting period estimates of old-age mortality (Qop) into single-year ones (Qo(t)) depends on the relationship between the over-time changes of Qo(t) and that of $Qo_2(t)$: the over-time changes of Qo(t) and that of $Qo_2(t)$ should be similar. To be more specific, if there is a peak (or bottom) in $Qo_2(t)$, there should also a peak (or bottom) in Qo(t). If $Qo_2(t)$ declines over time, so should Qo(t), and vice versa. To establish this relationship, the multiyear-period old-age mortality of the two-input-parameter MLT, Qo_2p, needs to be estimated.

In DCMD, Qo_2p can be defined using the period death rates Mp(60-64), Mp(65-69), and Mp(70-74), assuming the force of mortality to be constant within a age group:

$$Q_{o2p} = 1 - \exp[-5 \cdot \sum_x M_p(x)], \quad x = 60-64, \;\; 65-69, \;\; 70-74. \tag{1}$$

Further, the multiyear-period death rates can be calculated approximately as the average of single-year death rates in the period, if the population change slowly over time:

$$M_p(x) = \frac{\sum_t d(x,t)}{\sum_t p(x,t)} = \sum_t M(x,t) \cdot \frac{p(x,t)}{\sum_t p(x,t)} \approx \sum_t M(x,t)/T, \tag{2}$$

where *T* is the number of years included in the period, *d(x,t)* and *p(x,t)* are the numbers of death and population in age group *x* and year *t*, and *M(x,t)* is the single-year death rate of age group *x* and year *t*. Furthermore, the sum of single-year death rates can be calculated as

$$\sum_x M(x,t) = -\frac{Log[1-Q_{o2}(t)]}{5}. \tag{3}$$

Thus, Q_{o2p} can be calculated using (1)-(3).

Since $Q_{o2}(t)$ is determined by child and adult mortality, the basic relationship can be expressed as that the over-time changes of Qo(t) should be similar to that of $Q_{o2}(t)$, and can be written as

$$\frac{Q_{or}(t)}{Q_{op}} = \frac{Q_{o2}(t)}{Q_{o2p}}, \tag{4}$$

where Qor(t) denotes the single-year old-age mortality estimated from the relationship, and subscript "r" stands for relationship. Eq(4) indicates that the shapes of Qor(t) and $Q_{o2}(t)$ are similar, and the levels of Qo(t) and $Q_{o2}(t)$ are determined by Qop and Q_{o2p}. In other words, if there is a peak (or bottom) in $Q_{o2}(t)$, there will also be a peak (or bottom) in Qor(t). If $Q_{o2}(t)$ declines over time, so will Qo(t), and vice versa. Using (4), the relationship estimate of old-age mortality is derived as

$$Q_{or}(t) = \frac{Q_{o2}(t)}{Q_{o2p}} Q_{op}. \tag{5}$$

2.2. Smoothing the relationship estimates

Since the values of the Qop in two successive periods are often remarkably different, the changes in Qor(t) at the boundary of two successive periods are often discontinuous. These discontinuations can be eliminated using the local regression; and the smoothed estimates are denoted as Qos(t), where "s" stands for smoothed.

2.3. Utilizing the numbers of death in census years

For many developing countries, there are numbers of death by age (*d(x)*) collected from census (often within a 12-month period before the census date) or from vital registration for the calendar year of the census. Because the difference between a census date and the middle point of the 12-month period or the calendar year is 0.5 years or less, and because the age groups include 5 years, the census population can be adjusted to approximately refer the middle point by moving t_0 years along the survival line of the 5-year cohorts. Accordingly, the death rates of using data from annual death registration with a smaller than 0.5-year t_0, Modr(x), or of using data from deaths registered in census with a 0.5-year t_0, Modrc(x), can be calculated as:

$$\begin{aligned} M_{odr}(x) &= \frac{d(x)}{p(x)+t_0 \cdot d(x)}, \\ M_{odrc}(x) &= \frac{d(x)}{p(x)+0.5 \cdot d(x)}, \end{aligned} \tag{6}$$

where *p(x)* are the numbers of population in age group *x* at the census year. Subsequently, old-age mortality Qodr (using data from death registration, DR) or Qodrc (using data from deaths reported in census, DRC) are computed:

$$\begin{aligned} Q_{odr} &= 1-\exp[-5 \cdot \sum_x M_{odr}(x)], \\ Q_{odrc} &= 1-\exp[-5 \cdot \sum_x M_{odrc}(x)]. \end{aligned} \tag{7}$$

2.4. Weights

Using local regression, single-year values of old-age mortality are obtained as Qodr(t) and Qodrc(t). Since the data on death registration or reported in census can be used to make the estimates more reliable, the final single-year estimates of old-age mortality is estimated as

$$\begin{aligned} Q_o(t) &= w(t) \cdot Q_{os}(t) + (1-w(t)) \cdot Q_{odr}(t), \\ Q_o(t) &= w(t) \cdot Q_{os}(t) + (1-w(t)) \cdot Q_{odrc}(t), \end{aligned} \tag{8}$$

where w(t) are the weights.

The errors in Qos are mainly caused by the underreporting of census population and should be small when the underreporting rates of census population are small. On the other hand, besides the underreporting of census population, the errors in Qodr or Qodrc are also caused by the incompleteness of DR or DRC, because the numbers of death are required in calculating Qodr or Qodrc according to (6)-(7).

When the values of Qodr (or Qodrc) are lower than that of Qos, it implies that the DR or DRC is incomplete, even if there were no underreporting of census population. Because Qos and Qodr (or Qodrc) both depend on the data of census population, and because Qodr (or Qodrc) depends in addition on data of DR (or DRC) that are known to be incomplete, the Qos should be more reliable than Qodr (or Qodrc). Subsequently, the weights of Qos should be bigger than that of Qodr (or Qodrc).

When the values of Qodr (or Qodrc) are higher than that of Qos, it implies that census population are underreported (or the underreporting of census population is more than the incompleteness of the DR (or (DRC)). Because the Qos depends only on the data of census population that are known to be underreported, and because the Qodr (or Qodrc) depends also on the data of DR or DRC that are known to be complete (or less incomplete than the underreporting of census population), the Qos should be less reliable than Qodr (or Qodrc). Subsequently, the weights of Qos should be smaller than that of Qodr (or Qodrc).

These analyses, however, still cannot determine the values of the weights. When arbitrary is inevitable, we choose 2/3 as the bigger weight and 1/3 as the smaller weight, which is perhaps the simplest arbitrary:

$$\begin{aligned} &w(t) = 2/3, \quad average(Q_{os}(t)) > average(Q_{odr}(t), Q_{odrc}(t)), \\ &w(t) = 1/3, \quad otherwise. \end{aligned} \tag{9}$$

If there are no data on death for a census year t_1, it is nature to use Qodr(t_1)=Qodrc(t_1)=0, w(t_1)=1, and Qo(t_1)=Qos(t_1). However, when there are data on death at census year t_2 that is successive to year t_1, there are $w(t_1)=1$ and $w(t_2)=2/3$. To make w(t) change continuously between t_1 and t_2, dummy Qodr(t_1) or Qodrc(t_1) are set using the same death rates at year t_2, and w(t) is set to change linearly from 1 at t_1 to 2/3 or 1/3 at t_2. For other special situations, weights can also be different from the default ones.

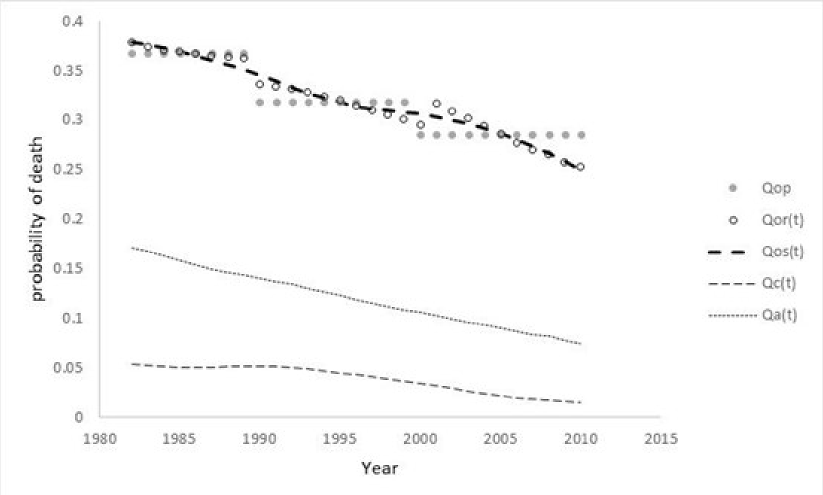
Finally, single-year life tables can be calculated using the three-input-parameter MLT (Li 2014), the Qo(t) expressed above, the Qc(t) provided by IGME, and the Qa(t) estimated by IHME. We calculated the single-year life tables for the 122 countries in DCMD, and we provided these life tables at www.lifetables.org.

3. Illustration

3.1. Period, relationship, and smoothed estimates

The estimates of Chinese women are chosen to provide below examples for illustration purpose. As can be seen in Figure 2, period old-age mortality (Qop) are displayed in grey circles, which are constant over time in the three periods between the censuses conducted in year 1982, 1990, 2000, and 2010. Relationship old-age mortality (Qor(t)) are shown in black-edged circles, which change over time slowly in1982-1990 when the decline of child mortality (Qc(t)) was slow and declines quickly in 1990-2010 when the declines of both Qa(t) and Qc(t) were faster.

Figure 2. Period, relationship, and smoothed estimates of old-age mortality, Chinese women.



Source: DCMD

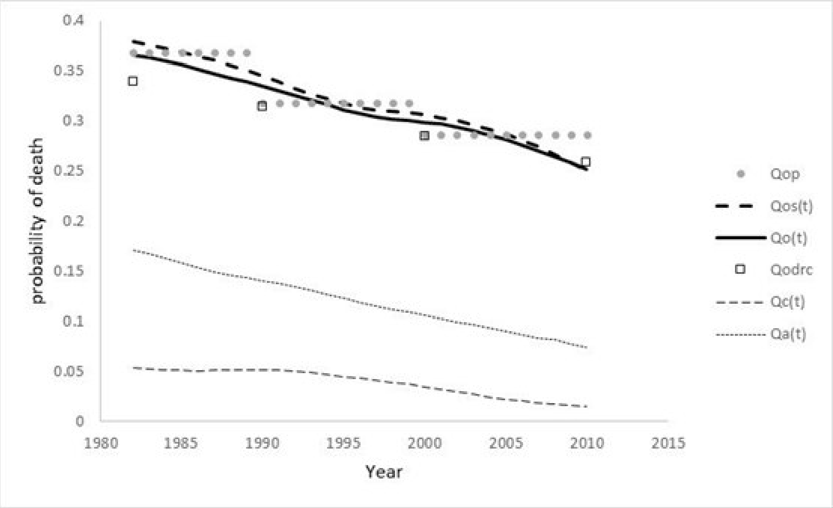
However, the changes in Qor(t) at the boundary of two successive periods, 1990 and 2000, are discontinuous. Local regression is used to eliminate the discontinuations and to produce the smoothed estimates Qos(t), which is described by the dashed curve in Figure 2. If there were no data on deaths at census years, Qos(t) would serve as the final estimates.

3.2. Data on deaths and final estimates

The censuses conducted in China also collected the numbers of death occurred within 12 (or 18) months before the census dates, which should be used to improve the reliability of estimation. These data, together with the data on population at census years, provided the additional estimates of old-age mortality (Qodrc), as are shown by the squares for the four census years in Figure 3.

Using (6)-(8), the final estimates, Qo(t), are obtained and shown as the solid curve in Figure 3. The values of Qodrc are closer to that of Qos(t) in more recent years, and make the final estimates Qo(t) differ more from Qos(t) in earlier years. Overall, the data on deaths improved the reliability of estimation.

Figure 3. Data on deaths and final estimates of old-age mortality, Chinese women.



Source: DCMD

4. Discussion

Compared to the numbers of deaths at child and adult ages, the number of deaths at old ages is the biggest (Figure 1) and, ironically, also the least reliable. This is because, for most developing countries, the numbers of old-age deaths are not estimated using observed data. They are based on the relationships between old-age and young-age mortality found mainly from developed countries.

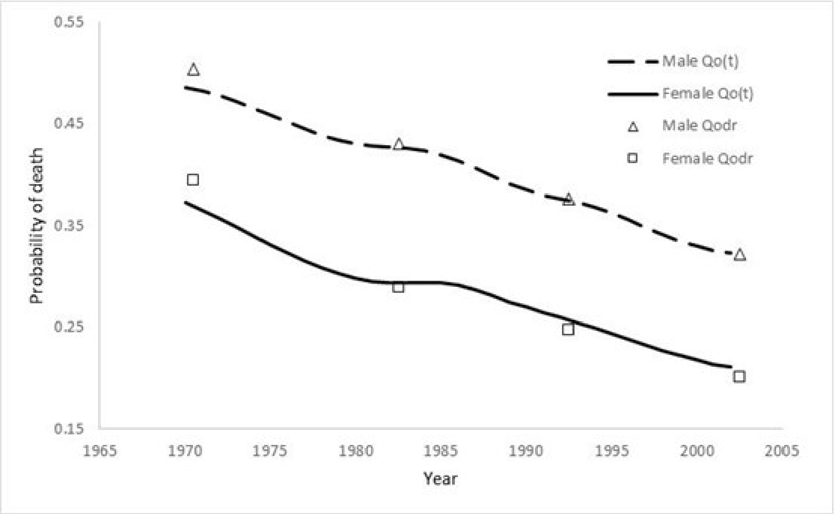
At old ages, migrants are rare comparing to deaths. Thus, census data on population by age and sex could be used to reliably estimate old-age mortality; and such data are available for almost all the countries of the world. Furthermore, in recent years new methodological developments have been made to use data on census population to estimate old-age mortality, and to extend the model life tables to utilize old-age mortality. The data availability and methodological developments led to the estimations of the multiyear-period life tables of the 122 countries in DCMD.

The multiyear-period life tables of DCMD are tested using the data of the Human Mortality Database (HMD, www.mortality.org), which contains 10-year period life tables. Applying the DCMD methods to the old-age population data of HMD after 1950, Li and others (2017) observed reductions of the root-mean-squared error (RMSE) in fitting old-age mortality for more than 70% of all the HMD countries, compared to using two-input-parameter MLT. In Figure 4, the location on the horizontal axis of each diamond describes the RMSE of using the two-input-parameter MLT to fit the old-age mortality of a country's male or female population over the six 10-year periods from 1950 through 2010; and the corresponding vertical-axis location indicates the RMSE of using that of the DCMD methods. In Figure 4, more than 70% diamonds are below the equal line, indicating that the DCMD methods improved the estimation of old-age mortality for more than 70% of the HMD populations.

Figure 4. Root-mean-squared errors in fitting old-age mortality for the populations in HMD

The multiyear-period life tables are converted into single-year life tables, using the method proposed in this paper. We calculated the single-year life tables for the 122 countries in DCMD, and we provided these life tables at www.lifetables.org . Here we choose the data of Chile, which is a developing country contained in DCMD, and is also included in HMD because its 'death registration and census data are virtually complete' (Human Mortality Database, 2018). In DCMD, single-year life tables are estimated for 1970-2002 for Chile. The values of old-age mortality estimated by DCMD methods are denoted as Qo(t). On the other hand, the values of old-age mortality are also calculated using the data from registered deaths and census population in the Demographic Yearbook (United Nations Statistics Division. 2013), which are denoted as Qodr. The Qo(t) and Qodr are compared in Figure 5.

Figure 5. Estimates of old-age mortality of Chile

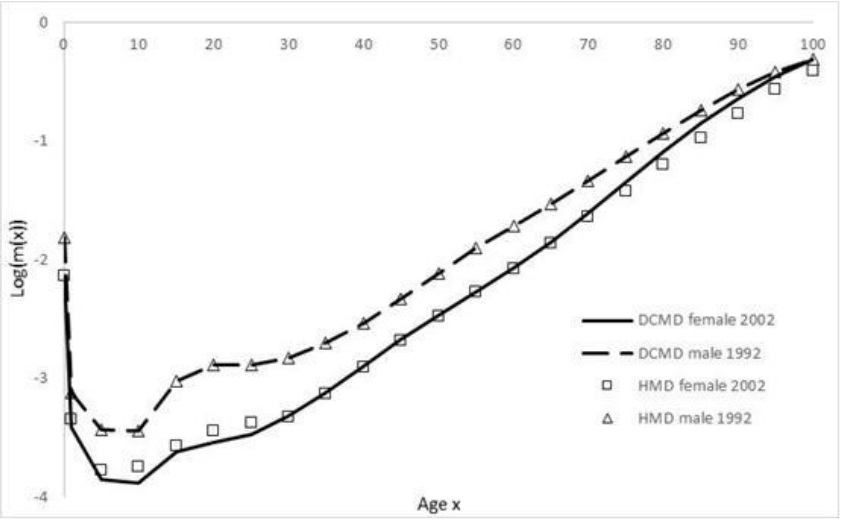


Sources: Qo(t) are from DCMD and Qodr from Demographic Yearbook

Besides at the middle years of the periods between two successive census years, we see in Figure 5 that, at census years, the values of old-age mortality from various sources are also almost identical.

The purposes of DCMD are not only to estimate old-age mortality, but also life tables. For Chile, the earliest life tables in HMD are for year 1992; while in DCMD the latest life tables are for year 2002. We choose the male life tables of 1992 and female life tables of 2002, from DCMD and HMD, to compare in Figure 6.

Figure 6. Estimates of the logarithm of death rate by age for Chile

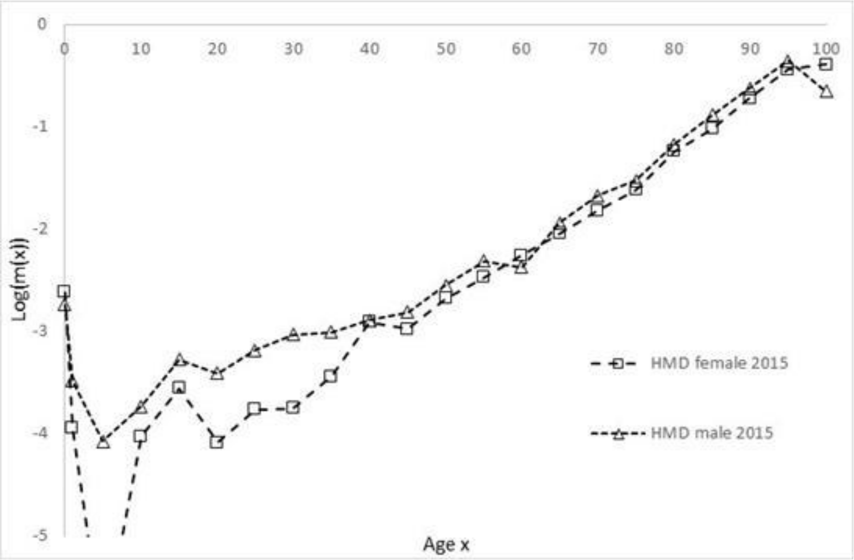


Sources: DCMD and HMD

We observe in Figure 6 that again, the logarithms of death rate by age from different sources are almost identical, indicating the accuracy of the DCMD single-year life tables for Chile. Checking the other few DCMD countries that are deemed to have reliable census and DR, for example Mauritius in Africa, and the Republic of Korea and Singapore in Asia, we observed that, like the case of Chile, the life tables of DCMD and various sources are also close. It is hard to explain these observations as coincidences. We believe these observations are the evidence supporting the accuracy of the DCMD single-year life tables. For many developing countries, DR is either unreliable or unavailable. Thus, it is difficult, if not impossible, to systematically test the accuracy of all the single-year life tables in DCMD. Nonetheless, readers may have some knowledge about the quality of DR and census for specific countries. They may make the above comparisons for these countries to obtain their own conclusions about the accuracy of the DCMD single-year life tables.

It is worth noting that the HMD life tables are calculated using death rates by 5-year age group, while in DCMD life tables are estimated using child, adult, and old-age mortality. The sound estimations of death rate by 5-year age group require quite large population size. For example, the population size of Iceland was about 330 thousand in 2015; and was not yet large enough to produce abridged death rates that are smooth over age, as can be seen in Figure 7 (Human Mortality Database, 2018). On the other hand, child and adult mortality are reliably estimated using 15 thousand households in the Demographic and Heath Surveys (2017).

Figure 7. Estimates of the logarithm of death rate by age for Iceland



Source: HMD

Thus, the closeness between the logarithms of death rate by age in Figure 6 is remarkable and, furthermore, may suggest that the DCMD methods could provide a solution for a longstanding problem in demography: estimating life tables for small populations.

In fact, there are four situations in estimating life tables: (I) large population size and reliable DR, (II) large population size and unreliable DR, (III) small population size and unreliable DR, and (IV) small population size and reliable DR. It is hard to say which situation is more important than another. The formal demography is competent for situation I, and the DCMD project aimed initially to deal with situation II. It turned out that, potentially, the DCMD methods could be a useful tool for situations III and IV, which are arguably an uncharted territory of demography.

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